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A THRESHOLD THEORY FOR SIMPLE DETECTION EXPERIMENTS[1]

R. DUNCAN LUCE

University of Pennsylvania

The two-state "high" threshold model is generalized by assuming that (with low probability) the threshold may be exceeded when there is no stimulus. Existing Yes-No data (that rejected the high threshold theory) are compatible with the resulting isosensitivity (ROC) curves, namely, 2 line segments that intersect at the true threshold probabilities. The corresponding 2-alternative forced-choice curve is a 45° line through this intersection. A simple learning process is suggested to predict S's location along these curves, asymptotic means are derived, and comparisons are made with data. These asymptotic biases are coupled with the von Békésy-Stevens neural quantum model to show how the theoretical linear psychometric functions are distorted into nonsymmetric, nonlinear response curves.

A classic postulate of psychophysics is that some stimuli or differences between stimuli never manage to affect the central decision making centers; others, of course, do. In a phrase, peripheral thresholds were assumed to exist. At least three types have been distinguished: absolute, difference, and detection. It is not, however, clear that there is any real difference among them. Absolute thresholds seem to be the same as detection ones except that the only noise is internal, and many difference threshold experiments differ from detection experiments only in the nature of the background stimulus, e.g., a pure tone or noise.

Recently the literal interpretation of the threshold postulate has been questioned by some detection workers, e.g., Swets (1961), Swets, Tanner, & Birdsall (1961), and Tanner & Swets (1954a) who have argued that thresholds, if one still wishes to call them that, are introduced only at the central decision level itself. What is important in this view is that the value of the "response threshold"—usually it is called something else, such as a decision criterion or cutoff—is not a fixed feature of the organism, but rather it is a parameter under the control of the experimental instructions, information feedback, payoffs, and other motivational factors. Two versions of such a threshold-free decision theory have been developed in detail. For signal detectability theory see Birdsall (1955), Green (1960), Licklider (1959), Peterson, Birdsall, and Fox (1954), Swets and Birdsall (1956), Swets, Tanner, and Birdsall (1955, 1961), Tanner (1955, 1956), Tanner and Birdsall (1958), Tanner and Norman (1954), and Tanner and Swets (1954a, 1954b). For the choice theory see Luce (1959), Restle (1961), Shepard (1957), and Shipley (1960, 1961). A number of experiments have been reported which

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agree with the main features of both theories. In addition to those reported in the above references, there are studies by Clarke, Birdsall, and Tanner (1959), Creelman (1959, 1960), Egan, Schulman, and Greenberg (1959), Green (1958), Green, Birdsall, and Tanner (1957), Shepard (1958), Swets (1959), Swets, Shipley, McKey, and Green (1959), and Veniar (1958a, 1958b, 1958c). Although the two theories differ conceptually, their predictions are so similar that it has been impossible as yet to decide between them.

In the course of evaluating signal detectability theory, a contrasting but equally explicit, sensory threshold model has been stated (Swets, 1961; Swets et al., 1961; Tanner & Swets, 1954a). It postulates that the threshold is well above the noise level. There is no doubt that this model is inadequate, and it has been concluded that if thresholds exist they must be so far down in the noise that the notion of a threshold ". . . is not a workable concept . . . [and] for practical purposes, not measurable" (Swets et al., 1961, p. 336). At least two sets of behavioral data do not jibe easily with this view.

First, there are studies, beginning with von Békésy (1930) and Stevens, Morgan, and Volkmann (1941), of the detection of energy increments of a pure tone background. Some of the results reported seem consistent only with a quantal (threshold) model. Although a number of people are dissatisfied with aspects of the experimental procedure and although the psychometric function has not always been found to be rectilinear as predicted by some quantum theorists, the recurring $n:(n-1)$ relation between the probability one and zero intercepts of the psychometric function has not been accounted for in any satisfactory way by a continuous, threshold-free model. The only published attempt that I know of is by Barlow (1961), and his rationalization seems completely ad hoc to me.

Second, Shipley (1961) has obtained some simultaneous detection and recognition data which indirectly suggest that detection thresholds exist. On each trial either a 1,000-cps tone in noise, a 500-cps tone in noise, or noise alone was presented, and the subject was required to decide whether or not a tone was present and, independent of his detection response, to attempt to recognize which it was. (Controls were run in which no recognition response was required and in which recognition was only required when the subject said a tone was present; there did not seem to be any interaction between the forced recognition responses and the detection responses.) If we separate the two detection responses, then we can ask how well he recognizes when he says he heard a tone as against when he said he did not hear one. If there really is a sensory threshold and if he reports no tone present only when the threshold is not exceeded, then there should not be any differential recognition of the tones on the no-detect trials. This is what happens, as can be seen in Table 1, for both the Yes-No and forced-choice designs.

This paper has two main purposes. First, a simple threshold model is described which appears to give as satisfactory an account of the response data as do the continuous detection theories. Second, a way is suggested to graft onto this sensory threshold model a decision process which predicts in some detail the biasing effects of information feedback, payoffs, and presentation probabilities. It is noteworthy that in conjunction with the present threshold model the usually

TABLE 1

PROBABILITY OF RECOGNITION CONDITIONAL ON THE DETECTION RESPONSE

Design	Presentation	Subject					
		1		2		3	
		Yes	No	Yes	No	Yes	No
Yes-No	500 cps	90	78	89	74	88	31
	1,000 cps	8	76	20	75	12	42
	noise	41	72	57	79	48	32
		Correct	Incorrect	Correct	Incorrect	Correct	Incorrect
Forced-choice	500 cps	88	43	87	63	87	43
	1,000 cps	14	47	25	63	13	41

Note.—These data from a simultaneous detection and recognition experiment are reported by Shipley (1961). On each trial of the Yes-No experiment either a 500-cps tone in noise, a 1,000-cps tone, or noise alone was presented, and the subject was required to respond whether or not a tone was presented and, independent of that response, to recognize which tone it was. In the table, the conditional probabilities (decimal points are omitted) that a presentation is recognized as the 500-cps tone are estimated separately for those trials when the subject said he detected a tone (Yes columns) and for those when he said he did not (No columns). On each trial of the two-alternative forced-choice experiment, either 500 cps or 1,000 cps appeared in exactly one of the two temporal intervals. The subject was required to state which interval contained the tone and which tone it was. The conditional probabilities that the presentation is recognized as the 500-cps tone are estimated separately for those trials when the subject chose the interval containing the tone (Correct columns) and for those when he was incorrect (Incorrect columns). Note that the conditional probabilities depend on the presentation in the Yes and Correct columns but are independent of the presentations in the No and Incorrect columns.

assumed expected-payoff model is completely unacceptable. Instead, a learning process is postulated. Thus, the present psychophysical theory is, in part, an asymptotic learning theory, as seems sensible.

YES-NO DETECTION EXPERIMENTS

It is generally agreed that one of the simpler detection experiments is the Yes-No design. On each trial unambiguous signals mark off a time interval during which either a background or the background plus a stimulus[3] is presented, and the subject is required to indicate whether or not he thinks the stimulus is there. Often the possible responses are said to be Yes and No, although in practice he usually selects one of two buttons to press. In many of the acoustic experiments, the background is white noise and the stimulus a pure tone, but this is not necessary. For example, in the von Békésy-Stevens quantal experiments, the background is a pure tone of one energy and the background plus stimulus is a tone of the same frequency but different energy. Nonetheless, I shall conventionally speak of the background as noise.

Let n denote a typical presentation of noise, s a typical presentation of stimulus plus noise, and Y the Yes and N the No responses. The basic data are the relative frequencies of a Y response given s, $\hat{p}(Y|s)$, and of Y given n, $\hat{p}(Y|n)$, which are assumed to arise from and therefore to estimate the true conditional response probabilities $p(Y|s)$ and $p(Y|n)$. With or without "hats," it is clear that $p(N|s) = 1 - p(Y|s)$ and $p(N|n)$

[3] In the signal detectability literature, the physical event to be detected by the subject has generally been called a "signal." As long as one is working with tones in noise and the like, this does not seem inappropriate; however, nothing in this theory restricts one to signals in this sense, so I have elected to use the more general term "stimulus."

$= 1 - p(Y|n)$. Our problem is, first, to explain how these two conditional probabilities relate to one another when we vary such experimental parameters as the a priori probability P of presenting s (and so $1 - P$ of n), the physical magnitudes of s and n, and the payoffs. The proposed answers, although far from complete, permit some experimental evaluation of the model. Second, given estimates of parameters from Yes-No data, we must try to account for the data from other experimental designs involving the same s, n, and subject.

We shall suppose that thresholds exist in the following sense. When either the noise alone or the stimulus plus noise is presented, the organism enters one of two hypothetical states denoted D and $\bar{D}$. A "detection observation" will be said to have occurred when he goes into State D and not to have occurred when he goes (or stays) in $\bar{D}$. These states are assumed to be internal to the subject and therefore cannot be directly observed in terms of behavior. Whether they can be studied by physiological methods is an open question that we need not discuss here. We do not suppose that the same state necessarily results whenever a particular stimulus is presented, but rather that the state entered is determined by a random process that is characterized by fixed probabilities for a given subject, stimulus, noise, and experiment. Just where the variability enters in is not specified by the theory. The underlying conditional probability model for these detection observations (not responses) is

Presentation probability	Presentation	Observation D	Observation $\bar{D}$
P	s	$q(s)$	$1 - q(s)$
$1 - P$	n	$q(n)$	$1 - q(n)$

In words, $q(n)$ is the true probability that noise alone generates a detection observation, i.e., that it "passes" the threshold, and $q(s)$, the true probability that stimulus plus noise generates a detection observation. We assume that $q(s) \geq q(n)$.

In the absence of data, one might have supposed that $P(Y|s) = q(s)$ and $P(Y|n) = q(n)$, but this cannot be because as a matter of fact the values of $\hat{p}(Y|s)$ and $\hat{p}(Y|n)$ depend upon at least P, the instructions, and the payoffs; and these differences are much too large and systematic to be ascribed to variability in the data. Evidently, then, the subject must convert some of the D observations into N responses or some of the $\bar{D}$ observations into Y responses, depending upon how he wishes to bias the outcome. On the assumption that the D observations are all indistinguishable, or, at least, that the s and n distributions of D observations are the same and that this is also true of the $\bar{D}$ observations, it is plausible that the bias involves responding "incorrectly" to some random fraction of the observations. If so, we obtain two different sets of equations depend-upon which bias is introduced:
if $p(Y|n) \leq q(n)$, then

$$\begin{aligned} p(Y|s) &= tq(s) \\ p(Y|n) &= tq(n), \end{aligned} \qquad [1]$$

or if $p(Y|n) \geq q(n)$, then

$$\begin{aligned} p(Y|s) &= q(s)+u[1 - q(s)] \\ p(Y|n) &= q(n) + u[1 - q(n)], \end{aligned} \qquad [2]$$

where $0 \leq t, u \leq 1$.

For the moment, we are not concerned about the actual values of the bias parameters t and u; rather we assume that any particular value can be made to arise, and we eliminate these unknowns from Equations 1 and 2 to obtain the dependence of $p(Y|s)$ upon $p(Y|n)$ with $q(s)$ and $q(n)$ as parameters:

$$p(Y|s) = \begin{cases} \left(\dfrac{q(s)}{q(n)}\right) p(Y|n), & \text{if } p(Y|n) \le q(n). \\ \left(\dfrac{1-q(s)}{1-q(n)}\right) p(Y|n) + \dfrac{q(s)-q(n)}{1-q(n)}, & \text{if } p(Y|n) \ge q(n). \end{cases} \qquad [3]$$

This equation describes a very simple function, namely, a straight line segment from $\langle 0, 0\rangle$ to $\langle q(n), q(s)\rangle$ and another from $\langle q(n), q(s)\rangle$ to $\langle 1, 1\rangle$, the two portions of which we shall speak of as the lower and upper limbs, respectively.

In the signal detectability literature the function relating $p(Y|s)$ to $p(Y|n)$ has been called a receiver operating characteristic or, more briefly, an ROC curve, but it seems more appropriate to call it an isosensitivity curve.[4] In that theory, it is truly a smooth curve. Examples of curves generated by detectability theory are shown in Figure 1 and of ones generated by Equation 3, in Figures 2 and 3.

The high threshold model discussed by Swets (1961), Swets, Tanner, and Birdsall (1961), and Tanner and Swets (1954a) is the special case of this one in which $q(n) = 0$, and so it consists only of the upper limb, i.e., of the line segment from $\langle 0, q(s)\rangle$ on the ordinate to $\langle 1, 1\rangle$. This is not a satisfactory summary of the data, but the two line segments of Equation 3 do about as well as any of the continuous theories with the same number of free parameters, namely, two. For example Swets, Tanner, and Birdsall (1955, 1961) report data on visual brightness for four subjects, where the payoffs were varied and P was held at 1/2. And Tanner, Swets, and Green (1956) report acoustic data on the detection of a 1,000-cps tone in white noise for two subjects, where the (symmetric) payoffs were held fixed and P was varied from 0.1 to 0.9 in steps of 0.2. In Figure 1, I have presented the data and detectability curves for one subject from each experiment, choosing in each case the subject that most favors signal detectability theory. All of the data and the curves of the present threshold model are shown in Figure 2 for the visual experiment and in Figure 3 for the acoustic experiment. Throughout the theoretical curves were fit by eye, because no optimal statistical procedure is known. (The theoretical crosses in Figure 3 will be discussed later.)

In evaluating the acoustic data, two facts are important. First, the $p(Y|s)$ coordinate of each data point is based upon a sample of 300 P, observations and the $p(Y|n)$ coordinate on 300 $(1 - P)$ observations. Second, successive pairs of points reading around the curve, were generated under identical experimental conditions. Thus, there can be little doubt that there is variability beyond the binomial associated with each observation point.

I would judge that the visual data slightly favor the threshold model and the acoustic data, the detectability model. Although different modalities may well involve different processes, neither set of data seems particularly conclusive. One feature of both sets, however, casts suspicion upon the present threshold model. The model says that the isosensitivity curve has a sharp corner which all too often seems to float free of these data points. Of course, this is exactly what would happen were the true function a

[4] I hope that the greater naturalness of this term, as compared with equisensitivity, will be adequate compensation for mixing Greek and Latin roots.

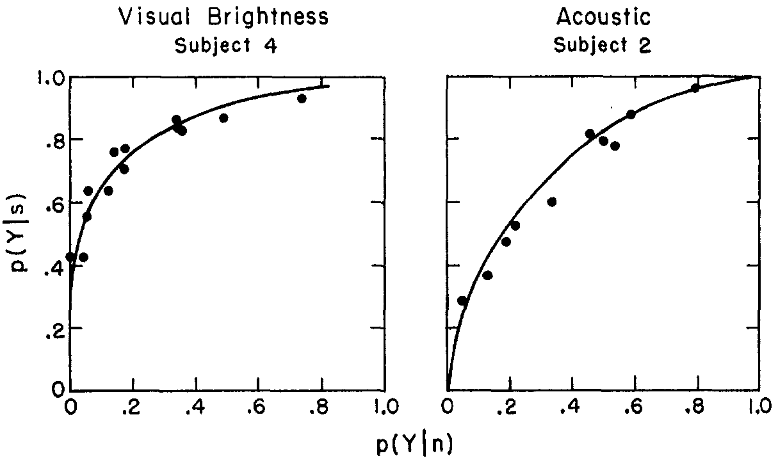

FIG. 1. Yes-No detection data and the corresponding theoretical isosensitivity curves derived from signal detectability theory. (The visual brightness data, reported by Swets et al., 1955, 1961, were obtained under the same stimulating conditions with a presentation probability of 0.5, but with different payoff matrices. The detection of a tone in noise data, reported by Tanner et al., 1956, was obtained under the same stimulating conditions with a fixed symmetric payoff matrix, but with different presentation probabilities.)

cornerless curve; hence, this threshold model does not deserve serious consideration unless the lonely corners are explained. A reason is suggested later.

Two-Alternative Forced-Choice Experiments

In the two-alternative forced-choice design two time intervals are defined and the stimulus is, and is known to be, in exactly one. Thus, the two presentations are the ordered pairs $\langle s, n\rangle$ and $\langle n, s\rangle$. The subject responds by saying which interval, 1 or 2, he believes to have contained the stimulus. Assuming the above threshold formulation, there are four possible observations, $\langle D, D\rangle$, $\langle D, \bar{D}\rangle$, $\langle \bar{D}, D\rangle$ and $\langle \bar{D}, \bar{D}\rangle$, of which two, $\langle D, D\rangle$ and $\langle \bar{D}, \bar{D}\rangle$ give the subject no indication of which response to make. It seems plausible, at least when the payoffs are not too extreme, that the subject should apply biases only to these two ambiguous cases. Thus, we assume that he always responds 1 when $\langle D, \bar{D}\rangle$ occurs, never when $\langle \bar{D}, D\rangle$ occurs, some proportion v when $\langle D, D\rangle$ occurs, and another proportion w when $\langle \bar{D}, \bar{D}\rangle$ occurs. If we assume that the observation probabilities in the two intervals are independent, then the probability of, say, a $\langle D, \bar{D}\rangle$ observation when $\langle s, n\rangle$ is presented is $q(s)[1 - q(n)]$ because $q(s)$ is the probability that the stimulus plus noise exceeds the threshold and $1 - q(n)$ is the probability that noise alone fails to exceed it. The other cases are similar, and they lead to

$$p(1\,|\,\langle s, n\rangle) = q(s)[1 - q(n)] + vq(s)q(n) + w[1 - q(s)][1 - q(n)] \quad [4]$$

$$p(1\,|\,\langle n, s\rangle) = q(n)[1 - q(s)] + vq(n)q(s) + w[1 - q(n)][1 - q(s)], \quad [5]$$

where $0 \leq v, w \leq 1$.

It follows by subtraction that

$$p(1\,|\,\langle s, n\rangle) = p(1\,|\,\langle n, s\rangle) + q(s) - q(n), \quad [6]$$

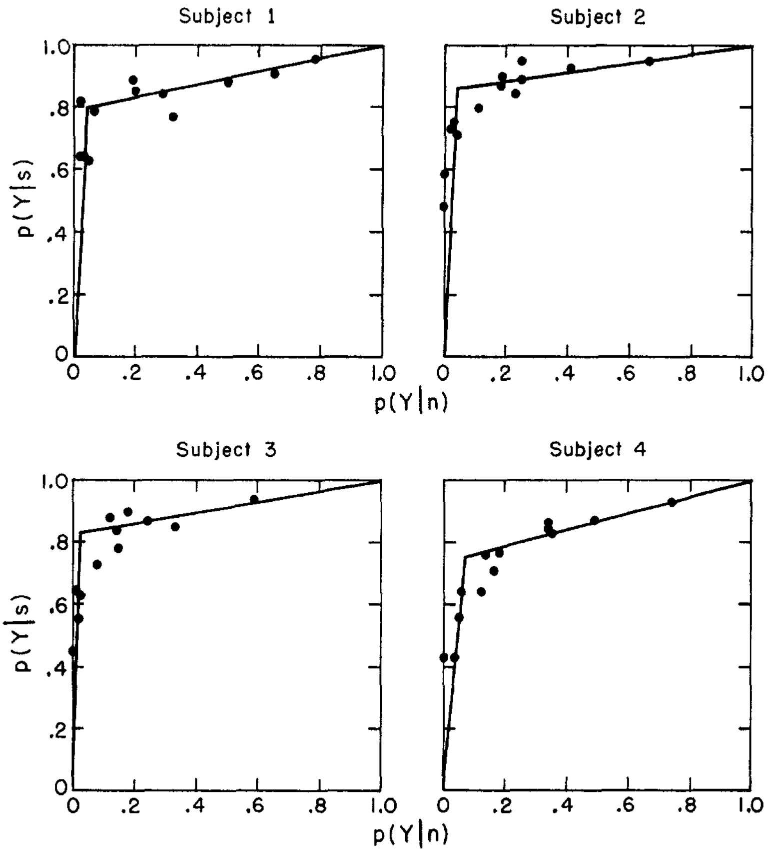

FIG. 2. Yes-No visual brightness detection data from Swets, Tanner, and Birdsall (1955, 1961) and the corresponding theoretical isosensitivity curves derived from a threshold theory. (Each coordinate of each point is based upon 200 observations.)

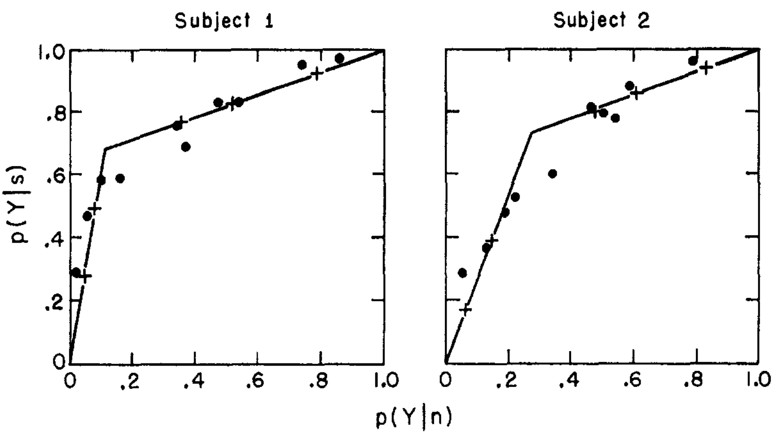

FIG. 3. Yes-No acoustic (tone in noise) detection data from Tanner, Swets, and Green (1956) and the corresponding theoretical isosensitivity curves derived from a threshold theory.

so the isosensitivity curve is a line segment with slope 1 running from $\langle q(n)[1-q(s)], q(s)[1-q(n)]\rangle$ to $\langle 1-q(s)[1-q(n)], 1-q(n)[1-q(s)]\rangle$. Thus, for example, if $q(s) = 0.9$ and $q(n) = 0.2$, the segment runs from $\langle 0.02, 0.72\rangle$ to $\langle 0.28, 0.98\rangle$.

It is also easy to see from Equations 4 and 5 that when $v = w = q(n)q(s)/\{q(n)q(s) + [1 - q(n)][1 - q(s)]\}$, then $p(1|\langle s, n\rangle) = q(s)$ and $p(1|\langle n, s\rangle) = q(n)$. That is, the two-alternative forced-choice isosensitivity curve passes through the point whose coordinates are the true threshold possibilities.

These last two remarks, coupled with the results about the Yes-No design, give a way to estimate the true threshold probabilities. Suppose for the moment that the several response probabilities are known. The point $\langle q(n), q(s)\rangle$ lies both on the 45° line passing through $\langle p(1|\langle n, s\rangle), p(1|\langle s, n\rangle)\rangle$ and on a line passing through $\langle p(Y|n), p(Y|s)\rangle$ and either $\langle 0, 0\rangle$ or $\langle 1, 1\rangle$, depending upon which limb of the Yes-No model is involved. Thus, the intersection of one of these two pairs of lines is the point $\langle q(n), q(s)\rangle$. The geometry is shown in Figure 4.

So far as I know, no empirical isosensitivity curves have been published for the two-alternative forced-choice experiment, so we cannot check our prediction that it is a straight line with slope 1. This prediction differs considerably from the curve—which is also symmetric about the diagonal from $\langle 0, 1\rangle$ to $\langle 1, 0\rangle$—predicted by signal detectability theory.

Asymptotic Learning

We turn next to the question of the values of the biasing parameters, t, u, v, and w. In the decision and choice theory models for these experiments, it has been customary to assume that the subject selects values for the biasing parameters so as to maximize his expected payoff. Let the payoff structure be

Presentation probability	Presentation	Response Y	Response N
P	s	o_{11}	o_{12}
$1 - P$	n	o_{21}	o_{22}

then if the subject is on the lower limb of the threshold model the expected payoff is

$$\begin{aligned} E(o) &= Pp(Y|s)o_{11} \\ &\quad + P[1 - p(Y|s)]o_{12} \\ &\quad + (1 - P)p(Y|n)o_{21} \\ &\quad + (1 - P)[1 - p(Y|n]o_{22} \\ &= t[Pq(s)(o_{11} - o_{12}) \\ &\quad + (1 - P)q(n)(o_{21} - o_{22})] \\ &\quad + Po_{12} + (1 - P)o_{22}. \end{aligned}$$

Because this equation is linear in t, the maximum occurs either at $t = 0$ or $t = 1$. A similar calculation for the upper limb yields either $u = 0$ or $u = 1$. Thus, the expected payoff model places the subjects at one of three points: $\langle 0, 0\rangle$, $\langle q(n), q(s)\rangle$, or $\langle 1, 1\rangle$. This is clearly wrong (see Figures 2 and 3).

Whether this prediction is wrong because of the threshold model or because of the expected payoff model is less easy to decide. One thing about the expected payoff model should be noted: knowledge of the two subject-determined conditional probabilities $p(Y|s)$ and $p(Y|n)$ is needed to calculate the values of the parameters. Certainly no one will claim that the subject "knows" these, even unconsciously, in a way that he can actually calculate expected values; more likely, he arrives at his biases by a process of adjusting to his experience—by learning. It is curious that no one has yet evolved a learning theory which, asymptotically, predicts the maximization of expected values. An alternative, and to my mind more

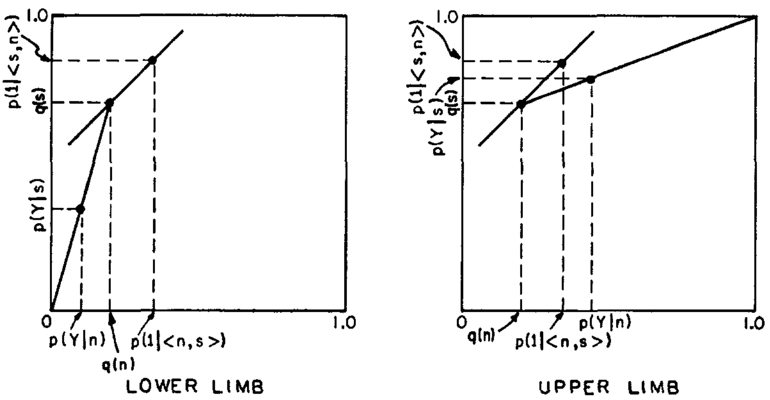

Fig. 4. The geometry relating the Yes-No isosensitivity curves, the two-alternative forced-choice isosensitivity curves, and the true threshold probabilities.

reasonable, tack is to postulate directly a learning process, preferably one that has already achieved some success in other areas, and to test its asymptotic predictions against behavior. This we do.[5]

Consider a subject who is operating on the lower limb of the Yes-No isosensitivity curve, and suppose that on Trial i the bias is t_i. What is the bias t_{i+1} on Trial $i+1$? Of the events occurring in Trial i, the only two that the subject should rationally take into account in modifying the bias are his observation, D or $\bar{D}$, and what he later learned the presentation to have been, s or n. If he is rational, he certainly should not let the response he made on Trial i or, for that matter, on any of the preceding trials influence his choice of bias. Because on the lower limb the bias only tells him how often to respond N to D observations, it seems clear that he should not change it when a $\bar{D}$ observation occurs. When a D observation results from an s presentation, the bias certainly should not be lowered, which would only decrease the Y responses, and it should not be increased when a D observation results from an n presentation. That is, we expect

$$t_{i+1}\begin{Bmatrix} \geq \\ \leq \\ = \\ = \end{Bmatrix} t_i \quad \text{if} \quad \begin{Bmatrix} \langle s, D\rangle \\ \langle n, D\rangle \\ \langle s, \bar{D}\rangle \\ \langle n, \bar{D}\rangle \end{Bmatrix}.$$

The exact nature of the transition is not obvious; however, a linear operator (Bush & Mosteller, 1955) is certainly one of the simplest possibilities and one that has received

[5] Conceptually, there is no special affinity between learning and thresholds, but in practice there are good reasons why it is easier to graft a learning mechanism on this threshold model than on the signal detectability model. In both there are three classes of events that a subject might use to control his learning: the hypothetical internal observations, his responses, and what he learns the presentation to have been. Both the first and third of these events form statistically stationary processes over trials, whereas the response probabilities are changing. Thus, if the learning process is dependent upon the responses, the resulting stochastic learning model is mathematically quite complex and I do not know how to analyze it. Although the other two classes of events do not have this particular complexity, the first can introduce a different kind. The internal observations that are assumed to occur in the detectability model take on values in a continuum, and so the learning model for this case must be continuous, and such models are not yet very well understood. The threshold model has the distinct advantage that there are only a small number of observations states, which results in a mathematically simple learning process.

considerable attention. So we assume[6]

$$t_{i+1}=\begin{Bmatrix}(1-\theta)t_i+\theta\\ (1-\theta')t_i\\ t_i\\ t_i\end{Bmatrix} \text{ if } \begin{Bmatrix}\langle s, D\rangle\\ \langle n, D\rangle\\ \langle s, \bar{D}\rangle\\ \langle n, \bar{D}\rangle\end{Bmatrix}. \quad [7]$$

Then

$$\begin{aligned}E(t_{i+1}|t_i) &= Pp(D|s)[(1-\theta)t_i+\theta]\\ &\quad +Pp(\bar{D}|s)t_i\\ &\quad +(1-P)p(D|n)(1-\theta')t_i\\ &\quad +(1-P)p(\bar{D}|n)t_i\\ &= t_i\{Pq(s)(1-\theta)\\ &\quad +P[1-q(s)]\\ &\quad +(1-P)q(n)(1-\theta')\\ &\quad +(1-P)[1-q(n)]\}\\ &\quad +Pq(s)\theta.\end{aligned}$$

Taking expectations over t_i and then the limit as $i \to \infty$ yields as the asymptotic expected bias

$$t_\infty = \lim_{i\to\infty} E(t_i) = \frac{q(s)}{q(s) + bq(n)}, \quad [8]$$

where

$$b = \left(\frac{1-P}{P}\right)\left(\frac{\theta'}{\theta}\right). \quad [9]$$

Similarly, we postulate the following learning process for the upper limb:

$$u_{i+1}=\begin{Bmatrix}u_i\\ u_i\\ (1-\theta)u_i+\theta\\ (1-\theta')u_i\end{Bmatrix} \text{ if } \begin{Bmatrix}\langle s, D\rangle\\ \langle n, D\rangle\\ \langle s, \bar{D}\rangle\\ \langle n, \bar{D}\rangle\end{Bmatrix}, \quad [10]$$

and a parallel calculation gives

$$u_\infty = \lim_{i\to\infty} E(u_i) = \frac{1-q(s)}{1-q(s)+[1-q(n)]b}. \quad [11]$$

According to Equation 9, the quantity b depends upon P and upon the two learning rate parameters θ and θ', which presumably in turn depend upon the payoffs. We have no theory for this dependence, so in general b will have to be estimated from the data or, as when we assume the learning rates to be equal for symmetric payoffs, an assumption will have to be made about θ and θ'. Clearly, if $\theta \neq 0$ and $\theta' \neq 0$, b ranges from 0 when $P = 1$ to ∞ when $P = 0$. At some point when P is varied the subject presumably changes from operating upon the upper limb to the lower limb. (See Figure 3 where P varies from 0.1 to 0.9.) We do not have a general theory for when this change occurs, but it seems plausible that it should be somewhere in the middle range of P values. If so, then u_∞ is bounded away from 0 to the extent that $q(s)$ is less than 1 and t_∞ is bounded away from 1 to the extent that $q(n)$ is greater than 0. Or translated back to the isosensitivity curve, the upper limb data points are prevented from being near the corner of the curve to the extent that $q(s)$ is less than 1 and the lower limb points are prevented from being near it to the extent that $q(n)$ is greater than 0. An examination of Figures 2 and 3 suggests that the data are consistent with this statement, which may explain why the corners seem to be isolated. It also suggests that information feedback need not always be beneficial in inducing subjects to yield up the desired information, in this case, the true threshold probabilities, as is often assumed by modern psychophysicists.

For the two-alternative forced-choice model, we assume essentially the same learning process, namely

$$v_{i+1}=\begin{cases}(1-\theta)v_i+\theta, & \text{if } \langle s, n\rangle \text{ and } \langle D, D\rangle\\ (1-\theta')v_i, & \text{if } \langle n, s\rangle \text{ and } \langle D, D\rangle\\ v_i, & \text{otherwise.}\end{cases}$$

[6] Although I will state the operators in terms of the bias parameters, they could equally well be stated in terms of the response probabilities because these probabilities are linear functions of the bias parameters.

A calculation similar to that for the Yes-No experiment yields

$$v_\infty = 1/(1+b). \qquad [12]$$

Similarly,

$$w_{i+1} = \begin{cases} (1-\theta)w_i + \theta, & \text{if } \langle s, n\rangle \text{ and } \langle \bar{D}, \bar{D}\rangle \\ (1-\theta')w_i, & \text{if } \langle n, s\rangle \text{ and } \langle \bar{D}, \bar{D}\rangle \\ w_i, & \text{otherwise} \end{cases}$$

yields

$$w_\infty = 1/(1+b). \qquad [13]$$

We note that $v_\infty = w_\infty$, as seems a priori reasonable, and that neither bias depends upon the underlying probabilities $q(n)$ and $q(s)$ as in the Yes-No experiment. We also note that if $P = 1/2$ and if the learning rates are equal, then the biases are symmetric in the sense that $v_\infty = w_\infty = 1/2$.

EMPIRICAL TESTS

To test the model, we have four sets of data, all collected on W. P. Tanner's equipment in the Psychophysical Laboratory, Electronic Defense Group, University of Michigan. Shipley (1961) ran each of three subjects in, among other conditions, the Yes-No and two-alternative forced-choice designs with $P = 0.5$ and with symmetric payoffs. Each condition was run twice with different stimuli, pure tones of 500 and 1000 cps. Each presentation, s or n in the Yes-No and $\langle s, n\rangle$ or $\langle n, s\rangle$ in the forced-choice design, occurred 800 times. Using the estimation scheme of Figure 4, values for $q(n)$ and $q(s)$ were obtained for both limbs. If either or both intersections lay outside the unit square, I selected the intersection of the 45° line through the forced-choice data point and the edge of the unit square as the final estimate. This incorrectly attributes all of the error variance to the Yes-No data point; however, because of the location of the two points, the forced-choice point undoubtedly has somewhat less binomial variance. Moreover, the learning process itself introduces added variance which more seriously affects our estimate of the Yes-No point than of the forced-choice one. Once $q(n)$ and $q(s)$ are estimated, then the theoretical location of the data points on the isosensitivity curves is determined by Equations 1 and 8 or 2 and 11 for the Yes-No experiment and by Equations 4, 5, 12, and 13 for the forced-choice experiment, provided that we know b. If we assume equal learning rates, then $b = 1$. The comparison between data and theory under that assumption is shown in Figure 5; it is surprisingly good, but unfortunately it does not permit us to decide which limb is being used. There is some suggestion that it may be the lower one, for in four of the six cases that and only that intersection is in the unit square, but this is far from conclusive.

Swets (1959) reported similar data on three subjects for several different signal to noise ratios. The plots are similar to those for Shipley's data; however, the predictions do not seem to be quite so accurate. In part this is due to the smaller sample sizes used by Swets.

Next, we have the acoustic data from Tanner, Swets, and Green (1956) which were presented in Figure 3. Again, because the payoffs were symmetric, we assume equal learning parameters, so b is determined by P. The predicted values, assuming that the $P = 0.1$ and 0.3 points are on the lower limb and that the rest are on the upper one are shown as crosses in Figure 3; recall that successive pairs of data points were collected under identical experimental conditions. The predictions seem satisfactory for Subject 1, but less so for Subject 2. Because no study has yet been made

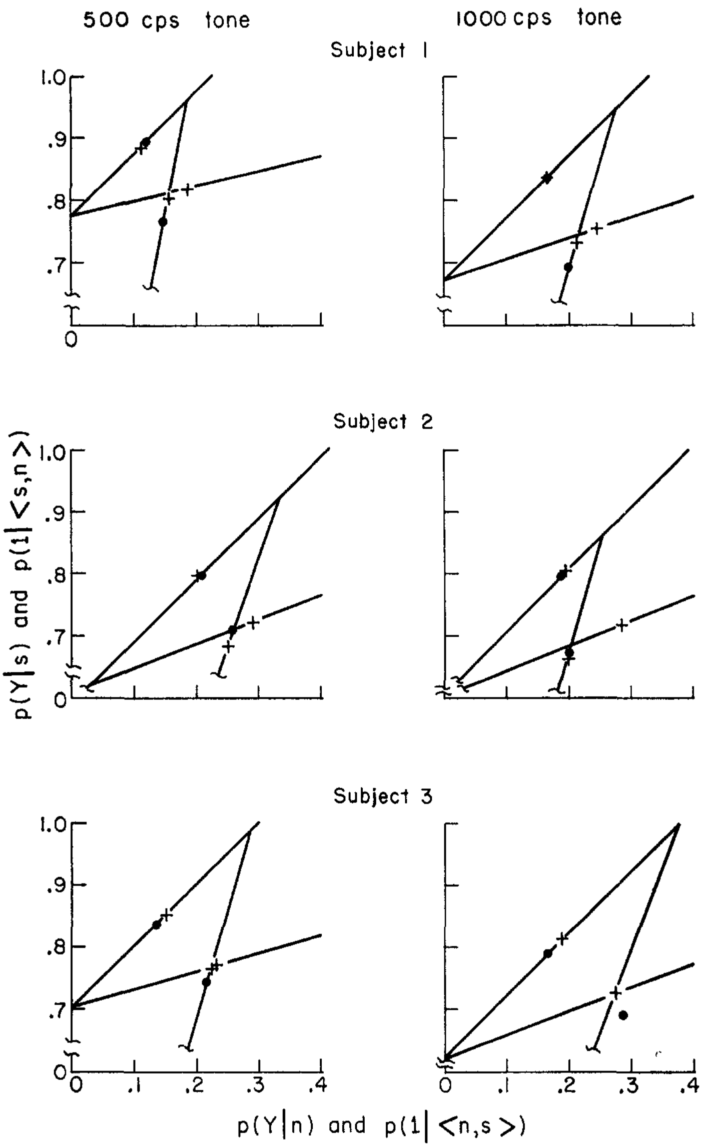

FIG. 5. Yes-No and two-alternative forced-choice acoustic (tone in noise) data reported by Shipley (1961). (The theoretical curves are from the threshold model and the predicted values—crosses—are asymptotic values derived from a linear learning process.)

of the learning process itself, I do not know how adequate the assumption $\theta = \theta'$ is, but to the extent that it is wrong errors are introduced into our predictions.

Comparable predictions for the visual data in Figure 2 are less easy to make because the isosensitivity curve was generated by varying the payoffs, not P. The only information that we have about the payoffs used are the numbers

$$\beta = \left(\frac{1-P}{P}\right)\left(\frac{o_{22} - o_{21}}{o_{11} - o_{12}}\right),$$

which are the relevant criterion quantities if one assumes that the subjects maximize expected payoffs and that they are described by the signal detectability model. If we assume that the learning rate parameter associated with s presentations, θ, is proportional to the difference of the two s payoffs, $o_{11} - o_{12}$, and that θ' is proportional to $o_{22} - o_{21}$, then

$$b = \left(\frac{1-P}{P}\right)\frac{\theta'}{\theta} = K\beta.$$

Thus, in addition to $q(n)$ and $q(s)$, there is the free parameter K to be chosen when fitting data. The values for $q(n)$ and $q(s)$ we get from Figure 2. Because it is reasonable that the two constants of proportionality relating θ and θ' to payoffs might be the same, I first tried using $K = 1$ to predict the responses. For three of the subjects this seemed satisfactory, but by trial and error I found that $K = 0.5$ is a much better choice for Subject 1. The results are shown in Table 2. Note the rather sharp break in both the observed and predicted values of $p_\infty(Y|n)$ as one moves from the lower to the upper limb, as indicated by the bold face vertical bars in the table, even though the changes in β are small in that region.

k-ALTERNATIVE FORCED-CHOICE EXPERIMENTS

The two-alternative forced-choice design can be readily generalized to one having k intervals, exactly one of which contains the stimulus. It is not easy to work out the response probabilities for any model, including this one, except under the assumption that

TABLE 2

ASYMPTOTIC LEARNING MODEL PREDICTIONS OF VISUAL DATA

Subject	$q(n)$	$q(s)$	K			β												
						8	8	6	4	2.5	2	1.5	1	.75	.75	.50	.25	.16
1	.05	.80	.5	$p(Y\|s)$	observed	64	64	82	79	—	63	85	77	89	84	88	91	96
					predicted	64	64	67	72	—	76	84	86	87	87	89	93	95
				$p(Y\|n)$	observed	2	3	2	6	—	5	20	32	19	29	50	65	78
					predicted	4	4	4	4	—	5	26	33	39	39	48	65	74
2	.05	.85	1	$p(Y\|s)$	observed	48	59	73	75	71	80	90	84	87	89	95	93	95
					predicted	58	58	63	69	74	86	86	87	88	88	89	91	93
				$p(Y\|n)$	observed	0	0	2	3	4	11	19	23	18	25	25	41	66
					predicted	3	3	4	4	4	12	14	18	22	22	28	42	52
3	.03	.83	1	$p(Y\|s)$	observed	45	56	—	64	73	65	78	90	84	87	88	85	94
					predicted	64	64	—	70	76	77	85	86	86	86	87	90	92
				$p(Y\|n)$	observed	0	2	—	2	8	1	15	18	14	24	12	33	59
					predicted	2	2	—	3	3	3	13	18	21	21	28	43	54
4	.07	.74	1	$p(Y\|s)$	observed	43	43	64	64	56	71	76	77	84	86	83	87	93
					predicted	42	42	47	53	60	77	78	79	81	81	83	84	91
				$p(Y\|n)$	observed	0	4	6	12	5	17	14	18	34	34	35	49	74
					predicted	4	4	4	5	6	18	22	27	32	32	40	42	66

Note.—Response proportions predicted by asymptotic learning model for visual detection data reported by Swets, Tanner, and Birdsall (1955, 1961). The vertical bold face line indicates the transition from lower to upper limb. Each observed proportion is estimated from 200 observations. Decimal points have been systematically omitted.

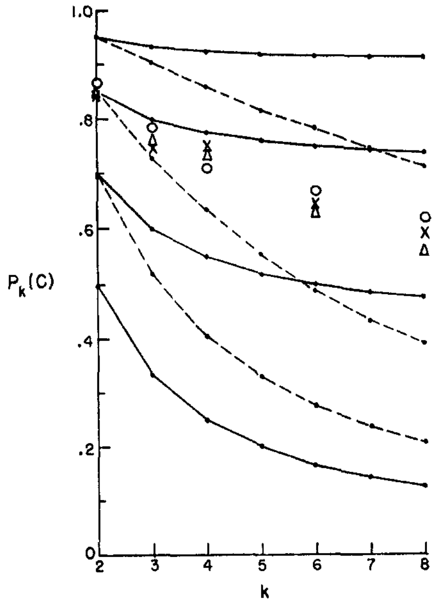

FIG. 6. Maximum and minimum curves of the proportion of correct responses in k-alternative forced-choice designs. (The data points for the detection of a tone in noise are from Swets, 1959.)

the asymptotic response biases are equal. In the two-alternative case, this means setting $v_\infty = 1/2$, which is what happens in the learning model if the learning rates are equal and $P = 1/2$. In general, it means that in any ambiguous situation the several possibilities are used equally often. The effect of this symmetry assumption is to make the probability of a correct response independent of the stimulus presentation. Correct reresponses can occur in the following ways: A response is always correct when the stimulus produces a D observation and the $k-1$ noise presentation produce $\bar{D}$ observations; it is correct one-half the time when s and exactly one n produce D observations, which can happen in $\binom{k-1}{1}$ ways; it is correct one-third of the time when s and exactly two n's produce D observations; etc.; and it is correct one-kth of the time when all intervals produce $\bar{D}$ observations. These are the only ways a correct response can occur, so

$$p_k(C) = \sum_{i=0}^{k-1} \left(\frac{1}{i+1}\right)\binom{k-1}{i} \times q(s)q(n)^i[1-q(n)]^{k-1-i} + \frac{1}{k}[1-q(s)][1-q(n)]^{k-1}$$

$$= \frac{1}{kq(n)}\{q(s) - [1-q(n)]^{k-1} \times [q(s)-q(n)]\}. \quad [14]$$

For $k = 2$, $p_2(C) = (1 + A)/2$, where $A = q(s) - q(n)$, whereas for $k > 2$, $p_k(C)$ depends upon both $q(n)$ and $q(s)$ and not just upon their difference. Thus, in contrast to other theories, $p_k(C)$ is not uniquely determined by $p_2(C)$. To get an idea of the freedom involved, assume A is fixed, then the limiting possibilities are when $q(n) = 0$ and $q(s) = A$, in which case

$$p_k(C) = [A(k-1) + 1]/k, \quad [15]$$

and when $q(n) = 1 - A$ and $q(s) = 1$, in which case

$$p_k(C) = \frac{1}{k}\left(\frac{1-A^k}{1-A}\right). \quad [16]$$

Typical examples of these bounds are shown in Figure 6. The data points are from Swets (1959); clearly they fall within the bounds.

Swets (1959) ran three other subjects in the Yes-No and the two- and four-alternative forced-choice designs. If we estimate the values of $q(s)$ and $q(n)$ from the Yes-No and two-alternative forced-choice data using the method of Figure 4, then we can predict what should be observed in the four-alternative forced-choice experiment.[7] Because it is not always clear from the Yes-No data which limb was

[7] I wish to thank J. A. Swets for providing me with the raw data to make these calculations.

TABLE 3

FOUR-ALTERATIVE FORCED-CHOICE EXPERIMENT

Subject	Signal to noise ratio in *db*	Estimated parameters				$p_4(C)$		
		Upper limb		Lower limb		Upper limb	Lower limb	Observed
		$q(n)$	$q(s)$	$q(n)$	$q(s)$			
1	9.4	13	77	25	89	67	62	62
	14.5	0	87	13	100	90	82	82
	16.6	0	89	11	100	92	85	88
2	9.4	19	72	33	86	58	53	52
	11.7	17	74	28	86	62	57	63
	14.5	7	78	29	100	75	64	75
	16.6	2	83	19	100	84	75	79
3	9.4	0	69	25	94	76	65	68
	11.7	7	80	27	100	77	66	73
	14.5	0	83	17	100	88	80	85
	16.6	0	92	8	100	94	88	90

Note.—Estimates of $q(n)$ and $q(s)$ from Swets (1959) Yes-No and two-alternative forced-choice data, and the predicted and observed values of $p_4(C)$. Each subject made 500 observations at each signal level in each experimental condition. Decimal points have been omitted on all the probabilities.

used, the calculations are reported for both limbs in Table 3. These predictions suggest that Subject 1 was operating on the lower limb; that Subject 2 was on the upper limb for at least the three most intense stimuli; and that the picture is not clear for Subject 3. It is certainly the case that one of the two predictions is always near the observed value.

Tanner, Swets, and Green (1956) report four-alternative forced-choice data for the same subjects whose Yes-No data are shown in Figure 3. Estimating $q(n) = 0.11$ and $q(s) = 0.68$ from the Yes-No data for Subject 1, we predict $p_4(C) = 0.63$; 0.60 was observed. For Subject 2, $q(n) = 0.28$, $q(s) = 0.74$, and we predict $p_4(C) = 0.51$; 0.56 was observed. In both cases, $p_4(C)$ was estimated from 297 observations.

DISTORTION OF THE PSYCHOMETRIC FUNCTION

A plot of the Yes-No detection probability versus a physical measure of the stimulus magnitude is usually called a psychometric function. For example, in the von Békésy-Stevens quantal theory, the theoretical function is 0 for all stimulus increments less than one amount, 1 for all increments larger than another, and a straight line between these two points. As no distinction has been made in the quantal literature between what we are calling $q(s)$ and $p_\infty(Y|s)$, it is not perfectly clear which function is meant. There is no question that in testing the theory, estimates of $p_\infty(Y|s)$ have been plotted against increment size, but an examination of the theory itself suggests that we should interpret it as referring to $q(s)$.

Assuming that the above learning model for biasing is correct, the relation between $p_\infty(Y|s)$ and $q(s)$ for the lower limb bias is obtained from Equations 1 and 8; it is

$$p_\infty(Y|s) = t_\infty q(s) = \frac{q(s)^2}{q(s) + bq(n)}.$$

Because $bq(n) > 0$, $p_\infty(Y|s) \leq q(s)$

on the lower limb, and its maximum value, $1/[1 + bq(n)]$, occurs when $q(s) = 1$. Similarly, Equations 2 and 11 yield the result for the upper limb:

$$p_\infty(Y|s) = q(s) + u_\infty[1 - q(s)]$$
$$= q(s) + \frac{[1 - q(s)]^2}{1 - q(s) + [1 - q(n)\, b}.$$

In this case $p(Y|s) \geq q(s)$, and its minimum value, $q(n) + [1 - q(n)]/(1 + b)$, occurs when $q(s) = q(n)$. If we suppose that $q(s)$ is a rectilinear function having $p = 1$ and $p = 0$ intercepts in 2:1 ratio and that $q(n) = 0.05$, then we get plots like those shown in Figure 7, where b is a parameter.

Once again, we are not sure when the subject switches from lower to upper limb biasing. It is clear that such a switch must occur, for when he is on the lower limb $p_\infty(Y|s)$ can never reach 1, no matter how intense the stimulus is.

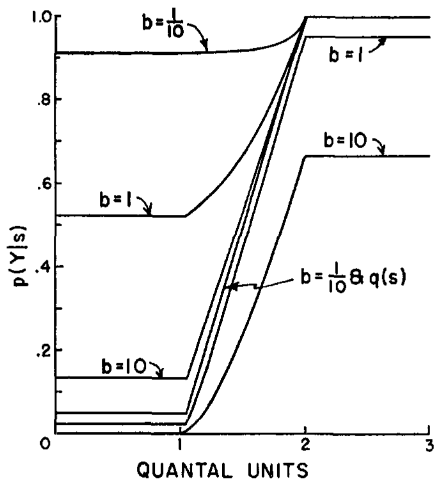

FIG. 7. Theoretical upper and lower limb psychometric functions when $q(s)$ is assumed to be rectilinear with a 2:1 ratio of intercepts, $q(n) = 0.05$ and $b = 1/10$, 1, and 10. (The upper limb curves are above, and the lower limb ones are below the $q(s)$ curve, which is indistinguishable from the lower limb $b = 1/10$ curve.)

The following hypothesis is currently under investigation and it appears to have some merit. In neural quantum theory (Stevens et al., 1941), those stimulus increments that cause zero and one quantum changes are assumed not to be detected, whereas those that cause changes of two or more quanta are. Let us suppose that this defines our states $\bar{D}$ and D, respectively. In addition to this assumption, let us postulate that the subject also uses the change in the number of quanta excited to decide which bias to use. Specificially, let us suppose that there is an integer $h \geq 0$, such that if the stimulus produces a change of fewer than h quanta, he imposes a lower limb bias, and that if it produces a change of h or more, he imposes an upper limb bias. We do not know what determines the choice of h, but presumably it depends in part upon instructions, presentation probabilities, and payoffs. In any event, we can see what sorts of psychometric functions result from different choices for h.

For $h = 0$, the subject always uses an upper limb bias; these are the upper functions shown in Figure 7. For all other values of h, there are regions of stimulation where either $h - 1$ or h new quanta are excited by the stimulus, and so the data will be a weighted average of the response curves resulting from lower and upper limb biases. The probability that h additional quanta are excited depends upon the probability that the energy residue of the background plus the stimulus exceed h quantal units of energy. In quantum theory it is usually assumed that the distribution of residues is (approximately) uniform, in which case we simply have the following rule: Let s denote the value of the stimulus in quantal units, if $s < h - 1$, then there is a lower

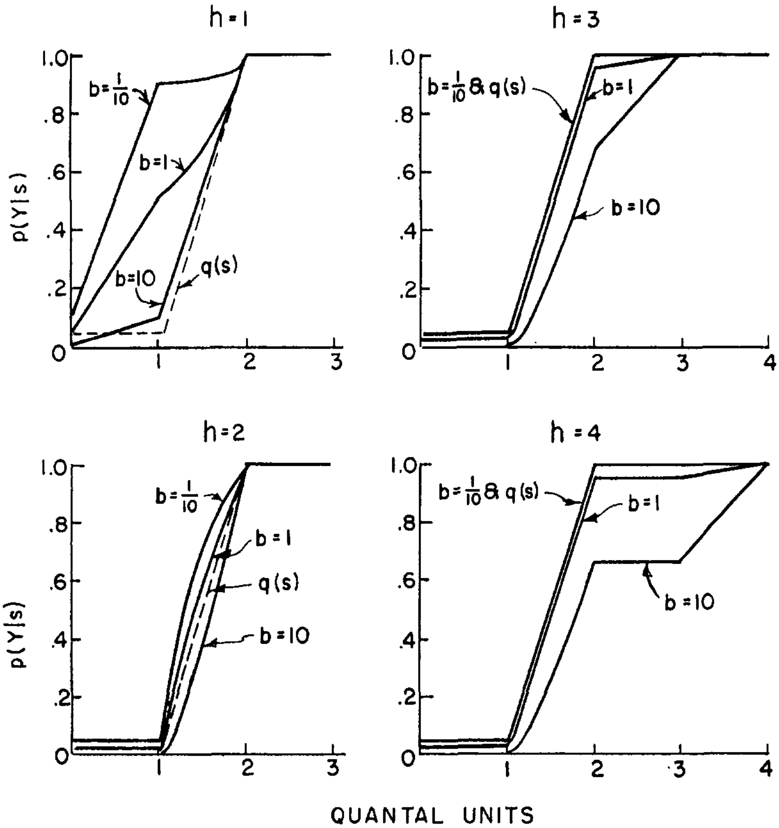

FIG. 8. Theoretical $p_\infty(Y|s)$ psychometric functions for different values of h when $q(s)$ is assumed to be rectilinear with a 2:1 ratio of intercepts, $q(n) = 0.05$, and $b = 1/10$, 1, and 10. (For $h = 3$ and 4, the $b = 1/10$ curve is indistinguishable from the $q(s)$ curve.)

limb bias; if $h - 1 \leq s \leq h$, then there is a lower limb bias with probability $h - s$ and an upper limb one with probability $1 - (h - s)$; and for $s > h$, there is an upper limb bias. Using this rule, typical functions are shown for $h = 1$, 2, 3, and 4 in Figure 8. For h larger than 4, the functions are just like those for $h = 4$, except that the right hand plateau extends over $h - 3$ quanta units.

CONCLUSION

The central conclusion of this paper is that there is at least one sensory threshold model for simple detection experiments which is not clearly wrong as judged by existing data. Four features of the model are noteworthy, of which two are really problems. First, the biasing effects on the response behavior that result from payoffs and presentation probabilities were treated as the asymptotic consequences of a linear learning process, not as the usually assumed maximization of expected payoffs which, when coupled with this threshold model, yields totally incorrect results. Second, the dependence of the asymptotic response probabilities on the probabilities of stimulus presentations is explicit, but the dependence upon the payoffs is given only in terms of learning rate parameters and so is implicit. A theory relating the learning parameters to payoffs is needed, but in the meantime sequential data should be collected and the parameters estimated directly as in ordinary applications of stochastic learning theory. Third, the threshold analysis of the Yes-No detection experiment led to an isosensitivity curve that consists of two distinct line segments, but no criterion was developed about which limb is in use during a particular experimental run. This leads to an ambiguity in the estimates of the true threshold probabilities, which continually proved to be a bothersome problem in analyzing data and evaluating the model. The data in Figure 3 and in Table 2 hint at the possibility that subjects may shift between the limbs when

$$\beta = \left(\frac{1-P}{P}\right)\left(\frac{o_{22}-o_{21}}{o_{11}-o_{12}}\right)$$

is in the neighborhood of 1.5 or 2, suggesting that it may be better to choose presentation probabilities and payoffs so that β lies well outside this transition region. In this way, we can be reasonably sure whether the subject is using a lower or upper limb bias. For example, if the payoff matrix is symmetric, one might use Ps in the neighborhood of 0.2 or 0.8, which places β in the neighborhood of 4 or $\frac{1}{4}$, respectively. Finally, the theoretical effects of this sort of biasing on the psychometric function were shown to be quite striking (see Figure 8). If this, or some such, model is correct, then fairly subtle analyses of response data are necessary in order to test adequately any theory of the psychometric function, such as the neural quantum theory.

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